Using picture story books to discover and explore the concept of equivalence

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The notion of equivalence is a very important concept for students and should be developed from a young age. This article demonstrates how students can deepen their relational understanding of the equals sign by exploring inequalities within a dice game based on familiar children's literature.

Introduction

Students deepen their relational understanding of the equals sign through exploring inequalities in this competitive dice game, built around the familiar fairy-tale *The Three Little Pigs* and *The Big Bad Wolf*. The activity can be adapted to different abilities by choosing more or less challenging dice combinations. The two follow-up investigations, based on the story *Who Sank the Boat?*, are intended to consolidate (Investigation 1), and further extend (Investigation 2), student understanding of the equivalence concept.

Context

Developing a relational understanding of the equals sign involves students interpreting this symbol as meaning 'the same as', rather than simply 'the answer'. It is a critical aspect of students' development in thinking mathematically that should be promoted as soon as students begin encountering number sentences (Karp, Bush & Dougherty, 2014). Such a relational understanding lays the foundation for algebraic thinking and promotes flexible representations of numbers (Molina & Ambrose, 2006).

For example, a relational understanding of the equals sign supports 'part-whole thinking', an important milestone in a young student's mathematical development which involves the student transitioning away from relying on counting-based strategies to using partitioning and compensation (Young-Loveridge, 2002).

This point is appropriately captured by Willis (2000), in her description of two Grade 1 students grappling with the number sentences 4 + 2 and 3 + 3. Whilst Sam understands that he can use his fingers to compute 4 + 2 = 6 and 3 + 3 = 6, for him these facts remain unconnected bits of knowledge. By contrast, Annie appears to grasp the connection between them, which suggests the foundations for an understanding of equivalence; in this instance that 4 + 2 = 3 + 3 = 6. In her own words:

> They both equal 6 because if you take one off the four and give it to the two, to make it three, then it is 3 add 3 or you could take one off the three and give it to the other three and make 4 + 2. That's why both have to be the same. (Willis, p. 32–33)

Many mathematics educators view the frequently narrow conception of the role of the equals sign in primary school classrooms as problematic. For example, Perso (2005) argues that students are conditioned to "do something now" or "find an answer now" whenever they encounter an equals sign (p. 214). She contends that this action-oriented, operational understanding of the symbol prevents students considering its relational aspect, which in turn impedes the development of algebraic thinking. She suggests a range of pedagogical approaches for attempting to address this misconception, including: using balance beams to visually play with concepts of equivalence, being exposed to practical worded problems which encourage the use of compensation strategies,

and using partitioning to encourage students to explore numerical equivalence in its symbolic form.

Despite its importance, developing this relational understanding of the equals sign can be extremely challenging, even when a teacher spends considerable time exploring the concept in the classroom (Seo & Ginsburg, 2003). One possible means of laying the foundation for a deeper understanding of equivalence may be to provide students with opportunities to discover this relational meaning of the equals sign. This discovery can be promoted through juxtaposing the concept of equivalence with the concept of inequality (and the corresponding inequality signs) early in a student's mathematical development (Russo, 2015).

This article will introduce a competitive dice game, built around the familiar fairytale, *The Three Little Pigs* and the *Big Bad Wolf,* designed to foster this discovery process. The article then outlines two follow-up investigations based around the text *Who Sank The Boat*? The first investigation provides students with a further opportunity to explore and consolidate the concept of equivalence using a different representation, specifically the balance-beam image suggested by Perso (2005). The second investigation further extends the concept of equivalence into a problem context involving proportional reasoning.

The game: *Three Little Pigs* versus *The Big Bad Wolf*

Teachers may wish to read a version of the fairytale prior to the activity in order to engage students before introducing students to the game.

Although the game is best suited to students aged from six to nine, older children could still benefit from the activity.

Setup

Students should play the game in pairs. The only equipment they need are various dice and some paper and pencil (or a whiteboard and whiteboard marker). The dice they should select depends on the age group and current ability level of students. The rules of the game are set out below:

- **Rule 1**: In pairs, one student plays the pigs and the other student the wolf.
- **Rule 2**: Dice are rolled, and students calculate their score for that role. For example, using the Years 3–4 dice, the player representing the pigs would sum the three 20-sided dice together, while the wolf would halve whatever number they rolled on their 10-sided 10s dice. The player with the higher score records the number sentence (using the greater-than or less-than sign), and earns a 'house'.
- **Rule 3**: First to five houses wins.
- **Rule 4**: If both players obtain the same score, they both record the number sentence (using the equals sign), and both earn a 'house'.

Teaching tips

It is recommended that rules 1, 2 and 3 be shared with students prior to them playing game. These three rules should be presented unnumbered on strips of card, and displayed prominently in the classroom (see Figure 1).

If students have difficulty during the game, the teacher should refer the students to the three game rules. However, rule 4 is best shared with

Table1. Suggested dice.

Suggested dice (with suggested operations in parentheses)				
	Three Little Pigs		*The Big Bad Wolf*	
Years 1–2	Three 6-sided dice (add)		20-sided dice (total)	
Years 3–4	Three 20-sided dice (add)		10-sided 10's dice (half)	

Figure 1. Introducing the first three rules.

the students only once the game has commenced or only after students raise the problem of players obtaining the same score.

More specifically, teachers should instruct students in using the greater-than or less-than sign appropriately in the pre-game introduction when the activity is launched. For younger students, consider introducing the application of the greater-than or less-than sign as the "crocodile always eating the larger amount". It is recommended, however, that teachers do not provide students with explicit instructions on what to do when the scores are the same. Ideally, the teacher should let the need to use the equals sign emerge from students' own reasoning, and explore this in more depth during the post-game discussion (see Figure 2).

If students ask about what to do in the case of a tie during the pre-game discussion, the teacher can respond something like "Hmmm I wonder if that will happen? If it does, let me know and we will decide what to do". Obviously, if, during the launch of the activity with the whole class, both players obtain the same score, the teacher may need to bring the discussion of rule 4 forward. The teacher will need three rounds or so to demonstrate the game to the students. It is worth noting that a tie is relatively unlikely to occur. (Using the dice recommended for older students, in a given round the probability of a tie is less than 2%.)

Some questions for guiding the post-game discussion appropriate for the first (and, depending on the age of the students, possibly second) time students play the game include:

- What was the score in your game?
- Did anyone have both players roll the same Wscore during a round?
- What did you decide to do? Did the game rules help?
- What new rule do we need to include in the game when both scores are the same?

Teachers working with older students (i.e., Years 3 and 4) can even get students to briefly work on this additional rule in pairs, record it and then share it with the class. Rule 4 can then be introduced on a strip of card, and displayed with the other rules in the classroom.

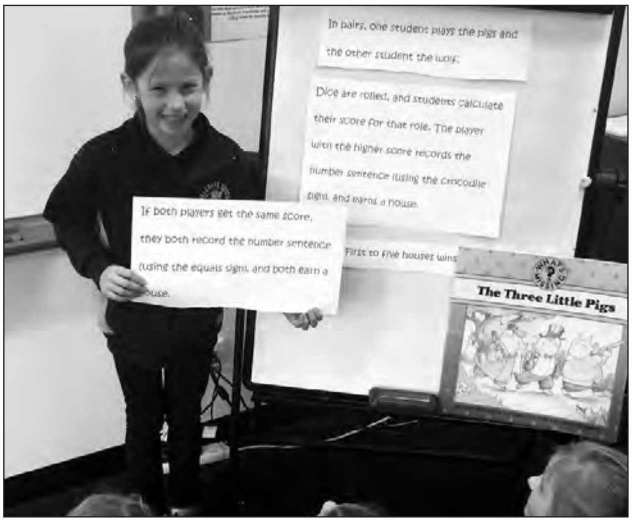

Figure 2. Discovering the fourth rule.

Get students to play the game again in subsequent sessions using all four rules. The game, even in this relatively simple format, can be revisited on several occasions. If you feel that students require further extension, the same basic game mechanism can support the use of more sophisticated strategies and concepts involving mental computation. For example, try playing with ten little pigs (ten 6-sided dice) vs three big bad wolves (three 20-sided dice); or, if exploring multiplication, three 6-sided dice that need to be multiplied together (for the pigs), vs a 10-sided 10s dice (for the wolf).

Example of a game

The game was played in a Year 3 and 4 composite class using the appropriate dice as previously described. Two Year 3 students Cada (pigs) and Samantha (wolf) began a game together. On the fifth round, when the players already had two houses each, Cada rolled a 20, 10 and 15 on her 20-sided dice, and Samantha rolled a 90 on her

ten-sided dice. After Samantha halved the number on her dice, the students realised that they had the same score (see Figure 3).

As this was the first time they had played the game, a great deal of excitement followed, and Cada yelled across the room “We got the same score, so we don’t know which way the crocodile sign should face. What should we do?! What should we do?!” The teacher asked “What do you normally do when two sides of a number sentence are the same? What sign would you use?” Samantha replied elatedly “The equals sign! They are the same! We use the equals sign!” The teacher replied that both students could record the number sentence, and both earn one house each. The need to use the equals sign only arose in around one-third of the games, however these instances provided a fascinating point of focus for the post-lesson discussion (Note that playing with the simpler dice, outlined for Grade 1 and 2 students, will result in the equals sign needing to be used more frequently).

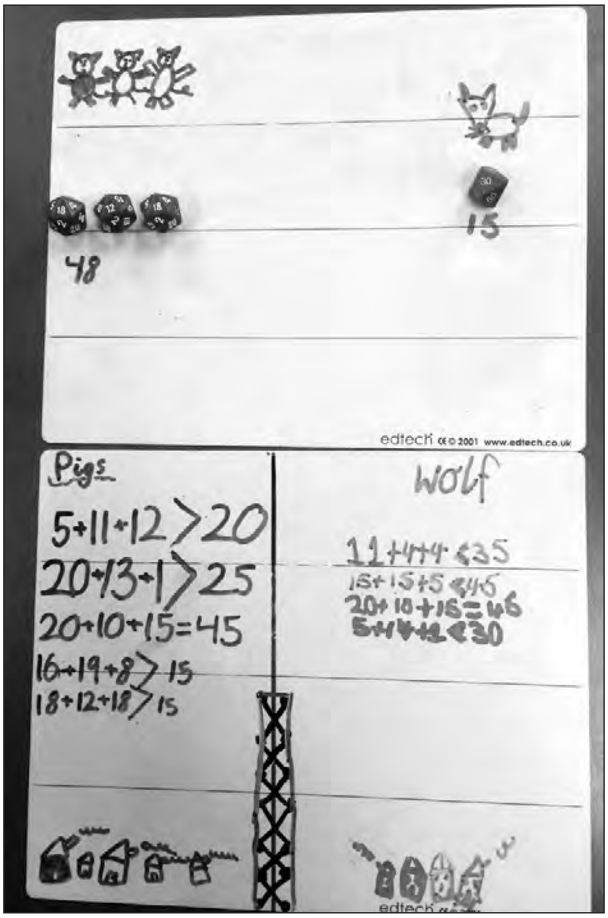

Figure 3. Example of a game: Cada (Pigs) vs Samantha (Wolf).

Consolidating and extending the concept of equivalence: *Who Sank The Boat?*

Context

Read the classic children’s story *Who Sank The Boat?* by Pamela Allen to the class, as a precursor to launching the following investigations.

The first investigation, “How can we balance the boat?”, is designed to consolidate students’ understanding of equivalence. The investigation explicitly incorporates Perso’s (2005) suggested balance beam representation of equivalence and allows students to tangibly and visually explore the concept. The open-ended nature of the first investigation, and its inclusion of an enabling prompt, supports differentiation and ensures it is a potentially suitable activity for students in Years 1 to 4 (Sullivan, Mousley, & Zevenbergen, 2006).

The second investigation, “How heavy is the mouse?”, is designed to build on the first investigation (hence students should have already undertaken the first investigation during a prior lesson). It is considerably more challenging and is suitable for older students (Years 3 to 5). It is designed to extend student understanding of equivalence through requiring students to apply the concept to explore interrelationships between unknown quantities. It involves proportional reasoning and more closely resembles a formal algebraic problem.

Figure 4. Plasticine ‘models’ of the cow, donkey, pig, sheep and mouse.

Investigation 1: How can we balance the boat?

Materials

- Paper and tape to create boats
- Playdough or plasticine to model the animals
- Pencils and paper to draw answers

Describing the problem

One of the reasons the boat stayed afloat so long is because the animals worked out how to balance their weights across the boat.

Can you find a way to get all five animals, including the mouse, to distribute their weight across the boat so that the boat is balanced and stays afloat? Here is some important information about the weight of the animals to help you with the problem:

- The cow weighs the same as the donkey (Cow = Donkey).
- The pig weighs the same as the sheep (Pig = Sheep).
- The cow and the donkey are both heavier than the pig and the sheep (Cow > Pig, Cow > Sheep; Donkey > Pig, Donkey > Sheep).
- The pig and the sheep are both heavier than the mouse (Pig > Mouse; Sheep > Mouse).
- See how many different ways you can solve the problem.

What do the students need to do?

- Create a boat using paper and tape.
- Model the animals using plasticine or playdough in accordance with the above information (see Figure 4).
- Use their animal models and paper boat to explore solutions to the problem (see Figure 5).
- Record their solutions by drawing them on paper as they discover them.

Advice for teachers

- Encourage students to work in pairs or groups of three to tackle the investigation. Mathematical reasoning and critical thinking can be supported by declaring that a solution may only be recorded when all group members agree that a particular configuration of animals would balance the boat. If agreement cannot be reached by the group on a particular configuration, the teacher should consider photographing it and exploring it further during the whole-class discussion (it may provide an opportunity to address a misconception or provide an example where there is genuine ambiguity about whether the boat would be balanced).
- There are theoretically infinite solutions to this challenge, some of which are displayed in Figure 5. However, the key insight into the problem is realising that the mouse needs to be exactly in the middle of the boat.

Enabling prompts for students

- If the mouse got onto the boat on his own, where would he need to stand to balance the boat?
- What if the mouse was in the middle of the boat? Would this help you solve the problem?

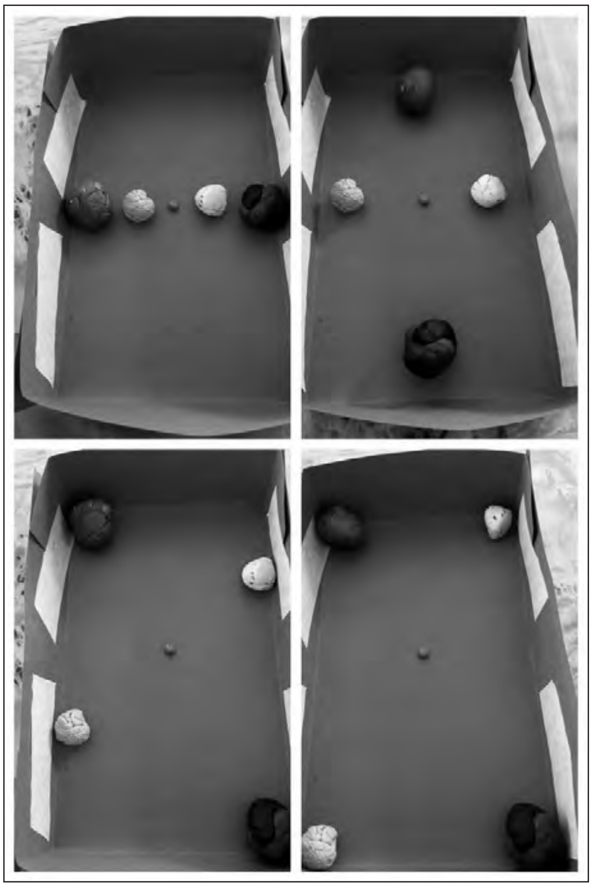

Figure 5. Some possible solutions to the 'How can we balance the boat?' investigation.

Investigation 2: How heavy is the mouse?

Materials

- Unifix blocks
- Pencils and paper to model answers

Describing the problem

Of course, the other reason the boat sank is because the combined weight of the animals was too heavy for the little row boat. You have been given some extra information about the animals' weights:

- The cow and the donkey are both twice as heavy as the pig and the sheep (Cow = 2 Pigs, Cow = 2 Sheep; Donkey = 2 Pigs, Donkey = 2 Sheep)
- The pig and the sheep are both five times heavier than the mouse (Pig = 5 Mouse; Sheep = 5 Mouse)

You have been told that the rowboat you have made can hold up to 100 unifix blocks before it sinks, so the combined weight needs to be less than this.

- Cow + Donkey + Pig + Sheep + Mouse < 100 unifix blocks

What is the maximum weight the mouse can be (in unifix blocks) to keep the boat afloat?

What do the students need to do?

Students can solve the problem however they like, however the teacher may wish to encourage students to physically model the problem with unifix blocks.

Advice for teachers

- The solution to the challenge is that the mouse can weigh 3 unifix blocks (i.e., 3 + 15 + 15 + 30 + 30 equals 93, which is less than 100). Although the challenge has only one solution, the extending prompt is designed to get students to generalise the relationships amongst the variables (i.e., the animal weights), and apply proportional reasoning. This process can be viewed as constituting an elementary form of algebraic reasoning (Perso, 2005).
- Although some concept of proportional reasoning is required to engage with the challenge, students should be encouraged to pursue the problem through trial and error. Combined with the enabling prompt, this should provide many students with a pathway into the problem.

Enabling Prompts

What if the mouse weighed one unifix block? How much would the pig and sheep weigh? What about the cow and donkey? How much weight would there be in the boat altogether?

Extending Prompts

Work out the maximum weight the mouse can be if the boat can hold up to:

- 200 unifix blocks
- 300 unifix blocks
- 500 unifix blocks
- 1000 unifix blocks
- 10000 unifix blocks

Conclusion

Building a deeper understanding of equivalence, and, in particular, grasping its relational aspect is both critical to developing number sense (Karp et al., 2014) and potentially very challenging (Seo & Ginsburg, 2003). It is suggested that playing the *Three Little Pigs* dice game, and undertaking the follow-up investigations using the text *Who Sank The Boat?* can help students to engage authentically with this critical concept.

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